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## Matching and Modeling of Multiple 3-D Disparity and Profile Maps

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# ABSTRACT

The thesis presents new methods for the matching and modeling of multiple 3-D data sets obtained from stereo or acquired by light striping. The concept of a profile map is introduced for the data acquired by light striping so that disparity maps and laser scanned data can be processed consistently as real valued images on parametric domains.

An iterative parametric point (IPP) algorithm is proposed for the simultaneous registration of multiple maps acquired from different viewpoints without known corresponding points when an initial registration is available. At each iteration, the corresponding points are determined directly on the parametric domains and the registration parameters are updated so that the mean of the squares of weighted distances between compatible corresponding points is minimized. Adaptive weighting is applied to the direction of the surface normal and to the distance between corresponding points, and the precision of the measuring is also incorporated into the weighting images. Edge areas are given less weight and the interpolation errors are leveled out in smooth areas for higher accuracy. The maps are registered hierarchically from low to high resolution for better convergence and shorter computing time.

The IPP algorithm is developed further for refining the calibration of the light striping system. The calibration includes a projective transformation between the image plane of the camera and the plane of the laser sheet, and the direction of the scanning with respect to the plane of the laser sheet. The proposed method is based on matching multiple profile maps registered previously according to an approximate calibration. The registration and calibration parameters can be refined simultaneously, but this requires a close initial estimate and rather complex object geometry. It is also possible to adjust several calibrations at the same time if the registration is known exactly.

The modeling of multiple registered maps includes techniques for merging compatible overlapping planar patches of different maps and tracing the borders of the merged patches on multiple domains. A technique is developed for the initial estimation of the parameters of a cylinder based on the cross product of non-parallel surface normals and on the intersection of a fan of planes.

The propagation of measurement and calibration errors to the registration parameters and to the modeled maps is analyzed carefully for two combined registration and modeling strategies when the object consists of planar patches. In the first strategy, the maps are registered simultaneously and the model computed afterwards while in the second one, the maps are registered sequentially against the model reconstructed up till then. The analysis is performed for various distance images that can be used in the registration including the Levenberg-Marquardt method and the method of unit quaternions to update the registration parameters.

The algorithms are formulated using image algebra and implemented efficiently using MATLAB software. Thorough testing with synthetic and real maps demonstrates the performance of the matching and modeling algorithms and illustrates the precision given by the two strategies. A comparison proves the superiority of the IPP algorithm to the iterative closest point algorithm as regards the accuracy, pull-in range, and computing time needed to update the corresponding points.

# PREFACE

This work has been done in the Institute of Photogrammetry and Remote Sensing at Helsinki University of Technology during 1993 - 1999. I wish to express my gratitude to Prof. Henrik Haggrén for the valuable discussions that have furthered my research. I would also like to thank Petteri Pöntinen, Keijo Inkilä, Min Gong, and the other people in the Institute of Photogrammetry and Remote Sensing who have contributed to my work.

I have held the post of a research scientist at Helsinki University of Technology since October 1994. I also greatly acknowledge the supports given by the Foundation of Technology in Finland, Heikki and Hilma Honkanen Foundation, and the Technology Development Centre in Finland in the beginning of this work.

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Finally, I would like to thank my parents for helping me in many ways during my life.

Espoo, March 2000

*Olli Jokinen*

# CONTENTS

# LIST OF THE PUBLICATIONS

The thesis consists of an overview and the following selection of the author's publications.

I Jokinen, O., Reconstruction of quadric surfaces from disparity measurements. *Applications of Digital Image Processing XVII* (Andrew G. Tescher, Ed.), Proc. SPIE 2298, San Diego, 1994, pp. 593-604.

II Jokinen, O. and Haggrén, H., Relative orientation of two disparity maps in stereo vision. *International Archives of Photogrammetry and Remote Sensing*, Vol. 30, Part 5W1, ISPRS Intercommission Workshop From Pixels to Sequences - Sensors, Algorithms and Systems (E. Baltsavias, Ed.), Zurich, 1995, pp. 157-162.

III Jokinen, O., Area-based matching for simultaneous registration of multiple 3-D profile maps. *Computer Vision and Image Understanding*, Vol. 71, No. 3, 1998, pp. 431-447.

IV Jokinen, O., Building 3-D city models from multiple unregistered profile maps. *Proceedings International Conference on Recent Advances in 3-D Digital Imaging and Modeling*, Ottawa, 1997, pp. 242-249, IEEE Computer Society Press.

V Jokinen, O. and Haggrén, H., Statistical analysis of two 3-D registration and modeling strategies. *ISPRS Journal of Photogrammetry and Remote Sensing*, Vol. 53, No. 6, 1998, pp. 320-341.

VI Jokinen, O., Self-calibration of a light striping system by matching multiple 3-D profile maps. *Proceedings Second International Conference on 3-D Digital Imaging and Modeling*, Ottawa, 1999, pp. 180-190, IEEE Computer Society Press.

The publications are referred to by Roman numerals in the overview.

# THE AUTHOR'S CONTRIBUTION

The author has written all the publications [I−VI]. In [II] and [V], the co-author provided some general ideas that initialized the work, but the author of this thesis developed the theory, implemented the algorithms, and carried out the experiments all by himself.

# 1 INTRODUCTION

The aim of the thesis is to develop new methods for the matching and modeling of multiple 3-D data sets acquired from different viewpoints. The focus is on data sets obtained from stereo in [I, II] and on data sets acquired by light striping in [III−VI]. The aim is that the data provided by these two measuring techniques could be processed consistently as real valued images using the image algebra described in [26].

Matching of 3-D data sets deals with estimating the parameters of certain coordinate transformations that bring the data sets into correspondence usually in the sense that a distance measure between the established corresponding points or extracted features is minimized. We consider first the registration problem where the unknown parameters determine the rigid body transformations between the 3-D data sets. Secondly, we investigate a matching approach for the self-calibration of the light striping system where the unknown parameters fix the relative orientations between the laser, camera, and object movement.

Modeling of 3-D data sets is concerned with interpreting the measurements as a set of mathematically expressed primitives. We study the segmentation of multiple registered data sets when the object consists of planar patches. We also investigate determining the borders of the planar patches in the overlapping areas. Furthermore, we address the problem of quadric fitting to segmented data for cones and cylinders. The registration task described above can also be considered as a part of the modeling.

In addition to the matching and modeling techniques, the aim is to derive the accuracy and precision of the estimated parameters and fitted model primitives. In precision estimation, the main effort is on different strategies that combine the registration and modeling tasks.

The research questions are stated more precisely and answered as follows. In Section 2, the techniques for 3-D data acquisition are described and the representation of a profile map is introduced for the data acquired by light striping. A novel algorithm is presented for the matching of multiple disparity or profile maps in Section 3. It is applied to solve the registration and calibration problems. The modeling issues after registration are considered in Section 4 and the propagation of errors is outlined for two strategies in Section 5. The performance of the methods is verified and the accuracy estimated in Section 6. Our contribution is summarized and the suggestions for future work are discussed in Section 7. The related literature is surveyed in [I−VI] so that this overview contains only a few additional and recently published references.

# 2 REPRESENTATION OF 3-D DATA

According to the image algebra, an image $\mathbf{u}$ is defined as the graph of a function $u$ from a coordinate set $S \subset \mathbf{R}^n$ to a value set $F$ (any semi-group). An element $(\mathbf{s}, u(\mathbf{s}))$ of $\mathbf{u}$ is called a pixel, where $\mathbf{s}$ is the pixel location and $u(\mathbf{s})$ the pixel value at location $\mathbf{s}$. The basic unary and binary operations on images are introduced in [26] and a short review of them is included in [III].

A stereo vision system provides the data in the form of a disparity map, which is a real valued image on a rectangular grid $S \subset \mathbf{Z}^2_+$ and can be regarded as a 2.5-D sketch of the scene [30]. The question is whether a similar representation can be considered for the data acquired by light striping. In the following, we first describe the disparity maps in terms of the image algebra and then introduce the profile maps in a positive answer to the representation question. In the subsequent sections, $\mathbf{u}$ denotes either a disparity map $\mathbf{e}$ or a profile map $\mathbf{j}$.

## 2.1 Disparity map

The horizontal (vertical) disparity is defined as a difference between the column (row) coordinates of the pixel locations of corresponding pixels in the left and right image of a stereo pair rectified to the normal case. In our system [14], the vertical disparities equal zeros and the horizontal disparities $e$ are determined for every location of the $N_1 \times M_1$ coordinate set of the original left image using feature-based matching and interpolation. To reduce the amount of data, we use every $s_n$th row and $s_m$th column of the coordinate set, starting from location $(1, 1)$. The disparity map is thus formally given by

$$\mathbf{e} = \{(\mathbf{s}, e(\mathbf{s})) \mid \mathbf{s} \in S = [1, \ldots, N] \times [1, \ldots, M]\}, \tag{1}$$

where $N = \lceil N_1/s_n \rceil$, $M = \lceil M_1/s_m \rceil$, and $\lceil \cdot \rceil$ denotes the smallest integer not smaller than its argument. Examples of real disparity maps appear in [I, Figs. 2-3] and synthetic ones in [II, Fig. 1]. The noise in the measured disparities can be characterized by the image of sample variances $\mathrm{Var}(\mathbf{e})$ obtained by repeating the measuring several times and using sample statistics.

Let $\mathbf{n}$ and $\mathbf{m}$ be two images on $S$ the values of which at $\mathbf{s}$ equal the row and column coordinates $n$ and $m$ of the coordinate set of the sparse left image at $\mathbf{s}$, respectively. In rectification, these images are transformed into $\mathbf{n}' = n_0 + s_r s_n (N - \mathbf{n})$ and $\mathbf{m}' = m_0 + s_r s_m (\mathbf{m} - 1)$, where the parameters $n_0$, $m_0$, and $s_r$ depend on the relative orientation of the original images. The right-handed $x, y, z$ coordinate system is fixed so that the origin is in the projection center of the rectified left image, the negative $z$-axis points into the direction of the sight, and the $x$-axis is parallel to the stereo baseline [I, Fig. 1]. Perspective projection yields for the object coordinate images

$$\begin{aligned} \mathbf{x} &= B\mathbf{m}' * \mathbf{e}^{-1}, \\ \mathbf{y} &= B\mathbf{n}' * \mathbf{e}^{-1}, \\ \mathbf{z} &= -BH\mathbf{e}^{-1}, \end{aligned} \tag{2}$$

where $H$ is the distance from the projection center to the rectification plane, $B$ is the base, '$*$' denotes the binary multiplication of images, and the inverse is in the sense of unary involution. The parameters $\mathbf{c} = [B\ H\ n_0\ m_0\ s_r]^T$ are assumed to be known from the calibration.

## 2.2 Profile map

The profile map provides a new way of handling the data acquired by light striping introduced in [III]. For each profile $p$ illuminated by the laser and for every $s_n$th row $i$ of the $N_1 \times M_1$ coordinate set of the image of the camera, the column index $j$ of the stripe seen in the image is measured with sub-pixel accuracy. The profile map is defined as a real valued image

$$\mathbf{j} = \{(\mathbf{s}, j(\mathbf{s})) \mid \mathbf{s} \in S = [1, \ldots, N] \times [1, \ldots, P]\}, \tag{3}$$

where $N = \lceil N_1/s_n \rceil$ and $P$ is the number of profiles recorded. The value $j(\mathbf{s}) = 0$ is assigned for locations outside the measurement coverage. The profile map is thus nothing but a convenient representation that supports the measuring technique. Its power in matching tasks is proven in the subsequent sections. Issues concerning multiple stripe observations on a single row and outliers are discussed in [III, VI]. The precision of the measuring is given by the image of sample variances $\mathrm{Var}(\mathbf{j})$ obtained by scanning the same view several times and using sample statistics [V]. Examples of real profile maps appear in [III, Figs. 4, 7; IV, Fig. 1; V, Fig. 2a] and synthetic ones in [VI, Figs. 2, 5]. The image of sample variances is illustrated in [V, Figs. 2b-2c].

In our light striping system [III−VI], the right-handed rectangular $x, y, z$ coordinate system is fixed so that the $xz$-plane is parallel to the plane of the laser sheet and the first profile measured is given the value $y = 0$. The object is moved stepwise with computer control in the direction of the negative $y'$-axis of a skewed $x', y', z'$ coordinate system defined so that the $x'$- and $z'$-axes coincide with the $x$- and $z$-axes, respectively [VI, Fig. 1]. We have

$$\begin{aligned} \mathbf{x}' &= (b_{11}\mathbf{i}' + b_{12}\mathbf{j} + b_{13}) * (b_{31}\mathbf{i}' + b_{32}\mathbf{j} + 1)^{-1}, \\ \mathbf{y}' &= s_p(\mathbf{p} - 1), \\ \mathbf{z}' &= (b_{21}\mathbf{i}' + b_{22}\mathbf{j} + b_{23}) * (b_{31}\mathbf{i}' + b_{32}\mathbf{j} + 1)^{-1}, \end{aligned} \tag{4}$$

where $\mathbf{i}' = s_n(\mathbf{i} - 1) + 1$ is a scaled image of the row indices, $\mathbf{p}$ is the image of the profile indices, $s_p$ defines the step size of the object movement, and the coefficients $\mathbf{b} = [b_{11} \dots b_{32}]^T$ determine the projective transformation between the image plane and the plane of the laser sheet. The skewed frame is rectified by

$$\begin{aligned} \mathbf{x} &= \mathbf{x}' + \mathbf{y}' x_0, \\ \mathbf{y} &= \mathbf{y}' \sqrt{1 - x_0^2 - z_0^2}, \\ \mathbf{z} &= \mathbf{z}' + \mathbf{y}' z_0, \end{aligned} \tag{5}$$

where $(x_0, z_0)$ is the orthogonal projection of the point $(0, 1, 0)$ of the $x', y', z'$ frame onto the $xz$-plane. The parameters $\mathbf{c} = [\mathbf{b}^T \; x_0 \; z_0]^T$ are solved and $\text{Cov}(\mathbf{c})$ estimated during calibration. The intrinsic parameters of the laser and camera are assumed known.

# 3 MATCHING OF MULTIPLE MAPS

In this section, a new iterative parametric point (IPP) algorithm is presented for the simultaneous matching of multiple maps without known corresponding points provided an initial estimate for the unknown parameters is available. The algorithm is applied to solve the registration problem in [II−VI] and to refine the calibration of the light striping system in [VI].

## 3.1 Registration of multiple maps

Assume that we have a set of disparity or profile maps $\mathbf{u}_k$ on $S_k$, $k = 1, \dots, L$, acquired from different viewpoints. After the calibration parameters $\mathbf{c}$ have been estimated, the images $\mathbf{x}_k$, $\mathbf{y}_k$, $\mathbf{z}_k$ can be computed according to Eqs. 2, 4, and 5. The $6(L - 1)$ unknown registration parameters, organized into a vector $\mathbf{a}$, determine the rigid body transformations between the first $x_1, y_1, z_1$ coordinate system and the other systems $x_k, y_k, z_k$ given by rotations $R_{1k}$ parameterized by three angles and translations $\mathbf{t}_{1k}$ for $k = 2, \dots, L$. Other parameterizations for the rotation are discussed, e.g., in [17].

The registration problem can be solved using external control points or features extracted from the maps. In [II], we use modeled features such as normals of planes, axes of cones and cylinders, and vertices of cones. The correspondences between the features are given manually. Our focus is, however, on area-based methods [II−VI] where the maps are matched as surfaces without known corresponding points and the feature-based method provides an initial registration for the matching algorithm.

The previous research on area-based registration is surveyed in [III] and more references are included in [V, VI]. As concerns the automation of the estimation of an initial registration, a method based on a viewpoint invariant representation of the data, a spin-image, is proposed in [18]. 2-D spin-images are generated for a set of oriented points having a position and direction given by the surface normal. A search is performed to establish point correspondences between oriented points in different views by comparing the associated spin-images through image correlation in overlapping areas. In [3], the initial registration is based on a hierarchical triangulation representation of the data sets. The idea is that the triangles in corresponding regions of different views should have similar areas and normal directions. The feature used in matching is a trihedron constructed from the normals of two adjacent triangles and the vector joining the centroids of the triangles. A set of possible initial registrations is generated searching and aligning pairs of compatible trihedra from different views. The initial estimates that provide largest overlaps between the views are considered most promising ones. In [9], the initial registration is obtained by matching points based on principal curvatures. The method is intended for recognizing objects that contain free-form surfaces. A heuristic search based on a pyramid representation of bicubic surface patches is also performed in [25]. In [8], a set of control points forming a triangular pattern in the domain of a data set is matched by trial and error against the points of another data set. Rigid constraints in the shape of the pattern restrict the search for corresponding control points in the other set after a candidate for one of the control points has been randomly selected. The estimate that maximizes the size of the overlapping area is chosen as an initial registration.

Our new computationally efficient method for the area-based registration is introduced in [II]. In [III], we improve weighting images proposed previously, present a simultaneous solution for multiple maps, and propose to apply hierarchical processing for better convergence. A summary of the algorithm appears also in [IV]. The precision of the registration parameters is derived from a careful analysis of error propagation in [V] and the high accuracy is proved using synthetic data in [VI]. The core of the method is the IPP algorithm described below in Section 3.3. Registration results given by the algorithm are illustrated in [II, Figs. 2-3; III, Figs. 5-6, 8-9; IV, Figs. 2-3].

## 3.2 Self-calibration of the light striping system

The calibration refers here to solving the parameters $\mathbf{c}$ that define the relative orientations between the laser, camera, and object movement in the light striping system. The related literature is referenced in [VI]. In most cases, the calibration is solved off-line using a separate calibration target with known dimensions.

Our contribution is to propose a new method for refining an approximate calibration on-line using the data only without any known calibration target [VI]. The idea is that multiple profile maps acquired from different viewpoints and already registered to the same coordinate system are matched having the calibration parameters as unknowns. The matching is based on the IPP algorithm.

The calibration result given by our method depends on the accuracy of the registration. On the other hand, an accurate calibration is important for the registration as systematic errors in the $(x_k, y_k, z_k)$ points may lead to a biased registration estimate. In [VI], it is shown that the highest accuracy is obtained when the registration and calibration parameters are refined simultaneously provided the initial values of the parameters are close to the true ones and the geometry of the object is complex enough for a successful convergence. It is also shown that several calibrations can be refined simultaneously, if the registration is known exactly. Calibration results given by the algorithm are illustrated in [VI, Figs. 3-4, 6-7].

The calibration parameters determine the 3-D structure of the scene. Our self-calibration method thus derives the structure from the motion of the object using shape correspondences. This can be viewed as an extension to solving the structure from motion problem, where only point or feature correspondences have been used previously.

## 3.3 Iterative parametric point algorithm

The new iterative parametric point (IPP) algorithm matches multiple 3-D data sets that can be represented as real valued images $\mathbf{u}_k$ on parametric domains $S_k$, $k = 1, \ldots, L$, such as disparity and profile maps, provided the transformation from the pixel (or map) coordinates to the $x_k, y_k, z_k$ coordinates given by the calibration of the system can be inverted. The algorithm refines the initial estimates of the registration parameters $\mathbf{a}$, calibration parameters $\mathbf{c}$, or both of them.

The key contribution is that the corresponding points are determined directly on the parametric domains $S_k$ by transforming the points from one map to the map coordinate system of another map [II; III, Fig. 2]. The method does not use the closest points in 3-D unlike most of the other methods based on the iterative closest point (ICP) algorithm [5]. A similar approach to ours has been proposed for range images in [6] published after [II].

The IPP algorithm proceeds hierarchically from low to high resolution and at each level of resolution, all the maps are matched simultaneously using compatible corresponding points in the overlapping areas [III]. The matching iterates two steps until convergence like the ICP algorithm. In the first one, the corresponding points are determined according to the current values of the parameters and in the second one, the values of the parameters are updated so that the mean of the squares of weighted distances between the corresponding points is minimized.

The distance image is given by $\mathbf{d}_{kl} = \mathbf{u}_{kl} - \tilde{\mathbf{u}}_l$, where the values of $\mathbf{u}_{kl}$ on $S_k$ equal the values of $\mathbf{u}_k$ when transformed to the system of $\mathbf{u}_l$ according to the current estimates of $\mathbf{a}$ and $\mathbf{c}$, and where the values of $\tilde{\mathbf{u}}_l$ on $S_k$ equal the interpolated values of $\mathbf{u}_l$ at the intermediate locations hit by the transformed image $\mathbf{u}_{kl}$. The corresponding pixel for $(\mathbf{s}, u_k(\mathbf{s}))$ is thus $(\mathbf{s}, \tilde{u}_l(\mathbf{s}))$, $\mathbf{s} \in S_k$. Bilinear interpolation is used in [II−V] while in [VI], it is replaced by a bicubic one almost everywhere in order to level out interpolation errors and thus to obtain a higher accuracy. Various other distance images that can be used in the matching are discussed in [V]. These images are related to the different ways of determining the corresponding points. The information from the parametric domain is used to accelerate the closest point search in a comparison to the ICP algorithm. This acceleration is the same as in [2].

The merit function to be minimized in the updating step of the IPP algorithm is given by

$$f(\mathbf{u}_1, \ldots, \mathbf{u}_L, \mathbf{a}, \mathbf{c}) = \sum_{l=2}^{L} \sum_{k=1}^{l-1} \mathbf{w}_{kl}^2 \bullet \mathbf{d}_{kl}^2 / K, \tag{6}$$

where $\mathbf{w}_{kl}$ are the weighting images between the corresponding points, '$\bullet$' denotes the dot product of images, $K = \sum_{l=2}^{L} \sum_{k=1}^{l-1} \mathbf{w}_{kl}^2 \bullet \mathbf{1}_k$, and $\mathbf{1}_k$ is a unit image on $S_k$ having ones at all locations. Within the overlapping areas, the weighting images adaptively reject incompatible matches in regard to the statistical distribution of the difference in the direction of the surface normal and to the statistical distribution of the distance between the corresponding points. The adaptive weighting has been proposed for the distance compatibility in [33] and we extend it to the direction of the surface normal in [III]. The interpolation errors are largest at locations near edges and they are weighted according to a decay function in [II−V]. For higher calibration accuracy, the edge locations are given zero weight in [VI] and the data sets are matched only in smooth areas so that the interpolation errors would be as equal as possible within the overlapping areas. Edge detection is based on the Laplacian image in [III−V] and it is improved for quadrics in [VI]. The precision of the data is introduced into the weighting images in [V] and it is changed to the precision of the distance between the corresponding points in [VI], following the work in [10]. In [10], it is assumed that the precision of the measuring is the same at all points while we extend the weighting to the case where the precision varies within the scene coverage. The Levenberg-Marquardt algorithm is applied to the simultaneous updating of the parameters so that $f$ in Eq. 6 is minimized.

# 4 MODELING OF MULTIPLE REGISTERED MAPS

This section considers the modeling of a set of registered maps $\mathbf{u}_k$ on $S_k$, $k = 1, \ldots, L$. It includes segmentation into parts and fitting appropriate primitives. The result is a set of modeled maps on $S_k$ denoted by $\hat{\mathbf{u}}_k$, $k = 1, \ldots, L$. Techniques are also presented for tracing the borders of the modeled patches on multiple domains $S_k$, $k = 1, \ldots, L$.

## 4.1 Segmentation

Segmentation of a single map deals with assigning labels to the pixel locations of the map so that locations belonging to the same surface patch have the same label. Most segmentation methods for 3-D data are based on the surface geometry and utilize the segmentation techniques developed for conventional gray level images [16].

In [IV], we apply a region-growing algorithm to segment a profile map $\mathbf{u}_k$ into patches which are planar in the $x_k, y_k, z_k$ frame. The direction of the surface normal is used as a criterion for growing at the first stage. Then, each location on a small segment is added to the neighboring large segment which is closest in the $x_k, y_k, z_k$ frame as measured by the distance from the point to the plane fitted to the data on the neighboring segment. This makes the segmentation more accurate near edges. In [I], we perform only a rough interactive segmentation of a disparity map into quadric patches.

In the case of multiple maps, the question is how to segment the overlapping areas where several maps contain data about the same patch of the object. This question can be addressed in two opposite ways. In the first approach, each map is segmented separately and then the compatible overlapping segments of different maps are merged [22]. In another approach [12], the data sets are first merged and then the whole data are segmented.

We consider the former approach in the case of planar patches in [IV, V]. Our achievement is a split and merge technique for the segmentation of ambiguous areas that may result from the misalignment of the maps. For each pair of maps $\mathbf{u}_k$ and $\mathbf{u}_l$, $k < l$, the segments of $\mathbf{u}_l$ that overlap several segments of $\mathbf{u}_k$ are split up in the overlapping area according to the segmentation of $\mathbf{u}_k$ and the compatible overlapping ones are merged. The algorithm is presented in [IV] and it is further improved in [V] concerning the order in which the segments are processed and using the confidence intervals of the estimated plane parameters to decide on whether two segments are compatible or not. The results of segmentation can be viewed from the reconstructed models in [IV, Fig. 4; V, Fig. 4].

## 4.2 Quadric fitting

Surface fitting to segmented data considers to estimate the parameters of the surface primitives that have been a priori chosen to represent the data on the segments. There exist plenty of different types of surfaces and our focus is on quadric surfaces. Quadric fitting is usually formulated as a nonlinear least squares problem, which is solved either using iterative methods for minimizing a nonlinear function or casting it as an eigenvalue problem which is solved directly and no approximate values for the parameters are needed.

The techniques for quadric fitting to segmented data are reviewed with emphasis on disparity data in [I]. Further references include [11], where plane fitting is addressed as an eigenvalue problem and quadric fitting is solved directly using the method of Lagrange multipliers. In [23, 24, 27], various iterative methods including the quasi-Newton, conjugate gradient, and simplex are compared for cylinder fitting. A close initial estimate is needed especially if the data cover only a part of the circumference of the cylinder. In [31], the reconstruction of objects having quadric patches is improved by incorporating geometric constraints that fix feature relationships between the patches. In [7], the parameters of a quadric are estimated from two quadratic curves fitted to the measured image coordinates of two stripes projected onto the object surface.

We apply the eigenvector method or equivalently, the total least squares method to plane fitting in [IV, V]. In cone and cylinder fitting in [I] and [19], we use the Levenberg-Marquardt nonlinear least squares method and choose the unknown parameters so that they have a direct geometrical interpretation such as the axis and vertex of a cone and the axis of a cylinder. Our contribution is focused on the initial estimation of the parameters of a cylinder [I]. The direction of the axis is estimated as a mean of a set of cross products of non-parallel surface normals and the location of the axis is fixed as an intersection of a fan of planes determined by the axis direction and surface normal vectors at the data points. We also stress the correct order of performing the rotations and the translation in rigid body transformations. The results of cylinder fitting are illustrated in [I, Figs. 5, 7].

The parameters of a model primitive are estimated from data transformed to the first $x_1, y_1, z_1$ coordinate system in case the support of the segment contains locations on several domains $S_k$ [IV]. The values of the modeled maps $\hat{\mathbf{u}}_k$ within the segment are computed so that the points $(\hat{x}_{k1}(\mathbf{s}), \hat{y}_{k1}(\mathbf{s}), \hat{z}_{k1}(\mathbf{s}))$, $\mathbf{s} \in S_k$, $k = 1, \ldots, L$, lie on the fitted surface. The modeling can be performed also in a single frame if the segment has support exclusively on a single domain. Moreover, an appropriate function can be fitted to the values $u_k(\mathbf{s})$ since these are corrupted by measurement errors only while the domain coordinates are noise-free. Such an approach is used for plane fitting to disparity data in [I]. In cone and cylinder fitting, the initial estimation of the axis line is performed in the $x_k, y_k, z_k$ system since the estimation is based on the object geometry that is usually not the same in the map coordinates. The results of modeling are illustrated in [I, Figs. 4-7; IV, Figs. 4-5].

## 4.3 Border tracing

Tracing the border of a surface patch deals with arranging the border locations in such an order that a closed line which does not intersect itself is obtained, when the ordered border points are connected in the $x_1, y_1, z_1$ coordinate system. The question is how to trace if the patch has support on more than one domain.

The case of a merged planar patch is considered in [22]. If the neighboring patches are also planar, the border of the patch is a poly-line consisting of vertices and non-occluded or occluded edges. The corresponding vertices and edges between the original patches that have been merged are first determined by matching vertex angles, edge lengths, and edge labels. The border of the merged patch is then traced out by following both borders around. “When neither edge is occluded then both edges are followed, when one edge is occluded the other is followed, and when both edges are occluded the outermost is followed” [22].

Our contribution is to propose a general algorithm for tracing the border of a merged patch applicable to patches of any type [IV]. The border locations are first determined independently on each map. Then the locations given by different maps are compared and the locations are removed which are actually inside the patch when all the views are considered. In case the same piece of the border has been measured from several viewpoints, only single border locations are stored for the tracing. The locations left are then traced on multiple domains by switching between the domains whenever the border locations end on a domain and continue on another one. The results given by the tracing algorithm are illustrated in [IV, Fig. 6].

After [IV], we have improved the definition of a border location so that there has to be at least one truly inside location also in the neighborhood of the border location. This makes the border smoother so that the tracing does not get stuck into thin ledges having the width of a single location. It is also checked that the new piece of line does not intersect the line traced so far. We may also take a couple of steps backwards during the tracing and select another location than previously.

# 5 STATISTICAL ANALYSIS OF TWO STRATEGIES

In this section, the propagation of measurement and calibration errors to the registration parameters and to the reconstructed model of the scene is analyzed in detail for two strategies.

## 5.1 Strategies for combined registration and modeling

Multiple maps can be registered sequentially or simultaneously, while our modeling algorithm proceeds only sequentially. A general discussion on the benefits of various registration strategies appears in [V]. The registration errors are usually more evenly distributed between the views in the simultaneous registration than in the sequential one. On the other hand, there exist direct methods to compute the motion between two data sets that can be applied in the updating step of the sequential registration while iterative methods are needed in the simultaneous case.

Two strategies are selected for a statistical comparison in the combined registration and modeling case in [V]. In strategy A, the registration is solved simultaneously and the model is built afterwards sequentially. In strategy B, each map $\mathbf{u}_l$ is registered either against all the previously registered maps $\mathbf{u}_k$, $k = 1, \ldots, l-1$, or against the model reconstructed up till then. In the first case, the model can be built after registration but in the second case, it is updated whenever a new map has been registered.

A previously unexplored question is how precise are the registration parameters and the model given by these strategies. We answer this question in the next section using error propagation techniques.

## 5.2 Precision estimation

The precision of the measuring varies within the scene coverage and it is characterized by the images $\mathrm{Var}(\mathbf{u}_k)$, $k = 1, \ldots, L$. In our statistical analysis [V], it is assumed that systematic errors in the measurements and calibration have been corrected and the precision of the calibration has been estimated in advance. It is further assumed that the data was correctly segmented, the type of the model primitive was correctly selected, and the registration has converged to a global minimum. The error propagation techniques yield then estimates for the precision of the registration parameters given by $\mathrm{Cov}(\mathbf{a})$ and for the precision of the modeled values given by $\mathrm{Var}(\hat{u}_k(\mathbf{s}))$, $\mathbf{s} \in S_k$, $k = 1, \ldots, L$.

The previous research on precision estimation related to the registration and modeling is reviewed in [V]. The registration and modeling steps are usually analyzed independently while our main achievement is to take into account all the error sources and dependencies related to the combined registration and modeling in the two strategies. In strategy A, errors in the data and calibration propagate to the registration parameters, and the registration errors propagate further to the reconstructed model. In strategy B, the registration errors accumulate when new maps are registered sequentially against the previously registered maps. In case the registration is performed against the model, the precision of the registration depends on the precision of the model reconstructed so far which, in turn, depends on the precision of the previous registrations. The errors in the registration and modeling thus propagate to each other by turns.

The precision estimation is based on the first order perturbation theory and on the standard rules of first order error propagation evaluated using the Jacobian matrices. In [V], we consider the case of profile maps and the calibration includes the parameters **b** of the projective transformation. We derive the precision estimates for various distance images used in the IPP and ICP algorithms. We also consider the Levenberg-Marquardt method and the method of unit quaternions when updating the motion parameters. Our analysis addresses the dependencies of the Hessian and Jacobian matrices on data carefully in the case of a Newton type minimization. The object consists of planar patches and we apply the total least squares method to plane fitting. The results are illustrated in [V, Figs. 3-6].

# 6 IMPLEMENTATION AND TESTING OF THE ALGORITHMS

This section verifies and illustrates the performance of the proposed methods in various test cases. The objective is to implement the algorithms so that the computations can be performed in a reasonable time.

## 6.1 Implementation

The algorithms are implemented using MATLAB software [21], which is a high-level programming language suitable for numeric matrix computation. The images defined on a rectangular grid can be considered as matrices and many image algebra operations are very similar to the built-in functions of MATLAB. As concerns the computing time, MATLAB performs much faster if the code has been vectorized using these functions.

Our contribution to the implementation issue deals with vectorizing the algorithms developed. This is realized firstly, when the algorithms are formulated using the image algebra and secondly, when the program code is written utilizing matrices and the built-in functions as much as possible.

In [III], it is shown how to vectorize the bilinear interpolation and the generalized convolution of an image and a translation invariant template. The estimation of the surface normals of a data set is changed from the eigenvector estimation in [I−V] to the one based on estimating the tangent vectors of two curves on the surface in [VI]. The latter method is vectorized and it performs about 42 times faster than the former non-parallel one. The speed-up is important in the self-calibration method since the surface normals change each time the calibration parameters are updated. Matching the surfaces hierarchically speeds up the processing, too.

The region growing is implemented to proceed in parallel in [IV]. Multiple regions are grown simultaneously and each region is grown at the same time to all the compatible neighbors. After two neighboring regions have been merged, the combined region continues growing with two seed locations.

The computing times are reported in [I, III, V]. It is shown that the registration and modeling tasks can be performed in the range of 10 minutes to a couple of hours, if the precision is estimated, too. The self-calibration method needs a bit more time than the registration. The computations are performed mainly in unix workstations including HP 9000/705 in [I, II], HP9000 Model 712/100 in [III, IV], and Digital Personal Workstation 433au in [VI]. The largest cases requiring much memory are computed in SGI Power Challenge hardware in [V] and in SGI Origin 2000 hardware in [VI].

## 6.2 Test results

The methods developed are tested with synthetic and real data. The object consists of planar and quadric surfaces in the synthetic and real disparity maps in [I, II] and in the synthetic profile maps in [VI]. The real profile maps include a casing box on a plate in [III] and a scale model of an urban area in [III−VI]. We use different maps over the latter object in [III, V, VI], but the same ones in [III, IV]. It is our contribution to show the techniques for generating synthetic profile maps in [VI]. This involves defining a scanning path, computing the observations as points of intersection of certain lines and the object, and dealing with occlusions properly. The precision of the measuring is illustrated in [V, Figs. 2b-c]. These images also show the areas that are difficult to measure by light striping.

### Registration

The first results on area-based registration are presented using two synthetic disparity maps in [II]. It is illustrated that the matching on the parametric domain works and the method can be used to refine the registration. On the other hand, the method based on the modeled features yields a more accurate estimate than the matching without most of the weighting images, if the modeling is successful [II].

The IPP algorithm for the registration of multiple profile maps is tested in two real data cases in [III, IV]. It is shown that after registration, the root mean of the squares of weighted distances between the established corresponding points in the overlapping areas is of the same order as the precision of the measuring. Note that this does not imply that the corresponding points and the registration estimate would be correct. However, the experiments support our understanding that matching the surfaces hierarchically helps in finding a global minimum for the merit function, when the initial estimate is relatively far away and the overlapping areas are large enough to allow the sub-sampling of the domains without essentially changing the characteristic geometry of the maps in the overlapping areas.

The precision of the registration parameters is evaluated through error propagation from the precision of the data and the calibration parameters in [V]. The experiments involve two to six profile maps scanned several times in the computer-controlled driveway. The precision $(\mathrm{Tr}(\mathrm{Cov}(\mathbf{a})))^{1/2} \approx 2$ is high enough to make the further analysis of error propagation meaningful.

The accuracy of the registration parameters is estimated using synthetic profile maps in [VI]. In a test case with two maps having an average noise level of 0.1 pixels in the $j_k$ values, the IPP algorithm results in the mean of the relative errors in the registration parameters of the order of 0.001% . A comparison to the ICP algorithm for point sets is presented in [VI]. Further test results and a comparison to the ICP algorithms for triangle sets and for bilinearly interpolated quadrilaterals [5] and to a variation of the ICP algorithm minimizing the distances from points to tangent planes at intersected points [4, 13] are shown in Tables 1-4. In the first two ICP ones, the closest vertex of the set of triangles or bilinearly interpolated quadrilaterals constructed from the values $\mathbf{r}_l(\mathbf{s})$, $\mathbf{s} \in S_l$, is first sought for each $\mathbf{r}_{kl}(\mathbf{s})$, $\mathbf{s} \in S_k$. The corresponding point is then the one which is closest to $\mathbf{r}_{kl}(\mathbf{s})$ on the six triangles or four quadrilaterals neighboring the closest vertex. In the case of quadrilaterals, the normal at a point on a quadrilateral is perpendicular to two non-parallel straight lines along the directions of bilinear interpolation and passing through the point. This gives two equations from which the point on the surface where the normal goes through a given point of the transformed view is solved numerically. In the variation of the ICP algorithm, the points of intersection between the surface normals at $\mathbf{r}_{kl}(\mathbf{s})$, $\mathbf{s} \in S_k$, and the surface mesh bilinearly interpolated from the values $\mathbf{r}_l(\mathbf{s})$, $\mathbf{s} \in S_l$, are first computed in a closed form. The corresponding points are then the closest ones on the tangent planes determined at the points of intersection. All the ICP algorithms have been accelerated as proposed in [5] and the translation and rotation have been separated in acceleration as suggested in [28, 29]. In the IPP algorithm, either everywhere bilinear or almost everywhere bicubic interpolation has been used in this comparison. The same weighting images have been implemented for all of the algorithms, but the weighting for the precision of the distance between the corresponding points has not been used here. The method of unit quaternions has been applied in the ICP algorithms and the Levenberg-Marquardt method in the IPP ones.

In Tables 1-4, $\tau$ is the deviation of the normally distributed noise added to the true registration parameters to generate an initial estimate while $N_s$ is the number of scans acquired from the same viewpoint and it relates to the noise level added to the data [VI]. A trial is considered successful if the mean of the relative errors in the estimated parameters is less than 1% (the threshold of 5% is used in [VI]). In Tables 1-2, the object consists of a paraboloid, cone, box, and background plane, and it is illustrated in [VI, Figs. 2, 4] while in Tables 3-4, the object is a randomly perturbed triangle set, a profile map of which is shown in [VI, Fig. 5]. In Tables 3-4, the figures denoted by 'x' were not computed since the number of successful trials had already decreased to zero at a lower value of $\tau$.

Table 1: Percentages of successful trials out of 20 (paraboloid, cone, box, and plane, $L = 2, N_s = 10$).

| $\tau$ | 0.01 | 0.1 | 0.5 | 1.0 | 1.5 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| IPP, bicubic | 100 | 100 | 100 | 100 | 100 | 85 | 75 | 80 | 75 | 55 |
| IPP, bilinear | 100 | 100 | 100 | 100 | 95 | 85 | 80 | 70 | 55 | 40 |
| ICP, triangles | 100 | 100 | 90 | 50 | 50 | 25 | 0 | 5 | 5 | 0 |
| ICP, bilinear | 100 | 100 | 100 | 50 | 20 | 20 | 15 | 10 | 0 | 0 |
| ICP, tangent pl. | 100 | 100 | 90 | 80 | 30 | 10 | 10 | 5 | 5 | 10 |

Table 2: Mean of the relative errors in **a** % (paraboloid, cone, box, and plane, $L = 2, N_s = 10$).

| | IPP | | ICP | | |
|---|---|---|---|---|---|
| $\tau$ | bicubic | bilinear | triangles | bilinear | tangent pl. |
| 0.01 | 0.0016 | 0.0050 | 0.018 | 0.016 | 0.014 |
| 0.1 | 0.00017 | 0.0054 | 0.043 | 0.096 | 0.053 |
| 0.5 | 0.00062 | 0.0044 | 0.16 | 0.14 | 0.077 |
| 1.0 | 0.0010 | 0.0056 | 0.27 | 0.29 | 0.23 |
| 1.5 | 0.0016 | 0.0066 | 0.28 | 0.42 | 0.33 |
| 2.0 | 0.00044 | 0.0032 | 0.26 | 0.60 | 0.51 |
| 3.0 | 0.00071 | 0.0058 | - | 0.28 | 0.17 |
| 4.0 | 0.056 | 0.0060 | 0.96 | 0.67 | 0.15 |
| 5.0 | 0.0011 | 0.0029 | 0.91 | - | 0.67 |
| 6.0 | 0.00057 | 0.0044 | - | - | 0.37 |

The results in Tables 2 and 4 clearly indicate that the IPP algorithms with bicubic and bilinear interpolation yield more accurate registration estimates than the ICP ones for triangle sets, for bilinearly interpolated quadrilaterals, and for distances from points to tangent planes at intersected points. When the initial estimate is close to the true one ($\tau \leq 0.1$), the accuracy given by the IPP algorithm with bilinear interpolation ranges from three to 17 times higher

Table 3: Percentages of successful trials out of 20 (randomly perturbed triangle set, $L = 2, N_s = 100$, 'x' means 'not computed').

| $\tau$ | 0.01 | 0.1 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| IPP, bicubic | 100 | 100 | 100 | 100 | 100 | 100 | 95 | 75 | 70 | 65 | 65 | 25 |
| IPP, bilinear | 100 | 100 | 100 | 100 | 95 | 100 | 90 | 65 | 65 | 50 | 55 | 55 |
| ICP, triangles | 100 | 95 | 0 | x | x | x | x | x | x | x | x | x |
| ICP, bilinear | 100 | 100 | 0 | x | x | x | x | x | x | x | x | x |
| ICP, tangent pl. | 100 | 95 | 0 | x | x | x | x | x | x | x | x | x |

Table 4: Mean of the relative errors in **a** % (randomly perturbed triangle set, $L = 2, N_s = 100$, 'x' means 'not computed').

| | IPP | | ICP | | |
|---|---|---|---|---|---|
| $\tau$ | bicubic | bilinear | triangles | bilinear | tangent pl. |
| 0.01 | 0.0029 | 0.0022 | 0.0057 | 0.016 | 0.0063 |
| 0.1 | 0.0024 | 0.0014 | 0.030 | 0.021 | 0.044 |
| 1.0 | 0.0045 | 0.0013 | - | - | - |
| 2.0 | 0.0017 | 0.0026 | x | x | x |
| 3.0 | 0.0014 | 0.0033 | x | x | x |
| 4.0 | 0.0011 | 0.0017 | x | x | x |
| 5.0 | 0.0015 | 0.0031 | x | x | x |
| 6.0 | 0.0029 | 0.0045 | x | x | x |
| 7.0 | 0.00075 | 0.00089 | x | x | x |
| 8.0 | 0.0024 | 0.0018 | x | x | x |
| 9.0 | 0.0029 | 0.00055 | x | x | x |
| 10.0 | 0.0037 | 0.0020 | x | x | x |

than obtained by the ICP one for bilinearly interpolated quadrilaterals. The estimates given by successful trials are also more centered round their mean in the IPP algorithms than in the ICP ones, which can be verified by computing the standard deviations of the estimates.

The IPP algorithms have larger pull-in ranges than the ICP ones as it is shown in Tables 1 and 3. For the object in Table 1, the ICP algorithms easily reject points on small quadric patches and converge to a local minimum where the planar areas fit perfectly, if the initial estimate is far ($\tau \geq 2.0$). This has also affected the accuracy figures of the ICP algorithms in Table 2. Changing the values of the constants in the adaptive thresholds ($\epsilon_i, \delta_i, i = 0, 1, 2$, in [III, p. 437]) does not improve the results essentially. However, the pull-in ranges of all the algorithms enlarge if the weighted sample mean $\nu_{kl}$ of the angles between the normals at matched points is replaced by $\max(\nu_{kl}, C_\nu)$ (where $C_\nu$ is a big constant given by the user) in the last term $\nu_{kl} \cdot \chi_{\geq \epsilon_2}(\nu_{kl})$ of the adaptive threshold in [III, p. 437, Eq. 17] and if the threshold for the distance compatibility is modified correspondingly. This modification follows the original adaptive scheme proposed in [33]. Further testing with the randomly perturbed triangle set has then shown that the IPP algorithm with bilinear interpolation gives at least 80% success for $\tau \leq 9$ and 55% success for $\tau \leq 13$ while the ICP algorithm for triangle sets yields at least 55% success for $\tau \leq 6$. These figures are obviously better than in Table 3.

In the IPP algorithm, the corresponding points are determined on the parametric domains of the maps. For profile maps, it follows that every two corresponding points lie on a fixed straight line in 3-D given by the intermediate row $i = i_{kl}$ of the image of the camera at view $l$ projected onto the plane of the laser sheet at the intermediate position $p = p_{kl}$ (cf. [III, Fig. 3]). These lines through different corresponding points are not parallel in general but well aligned in orientation. It is our understanding (stated without proof) that this combined with the adaptive weighting yields the larger pull-in range for the IPP algorithm as most of the correspondences have a rather consistent effect on the updating of the registration parameters. In the ICP algorithm, the closest points are established in any direction in 3-D and the different correspondences may be not so consistent among themselves as in the IPP one. The IPP algorithm also takes into account that there is noise only in the $j_k$ values of the profile maps while the ICP one, being more general, disregards this and handles all the $x_k, y_k, z_k$ coordinates equally.

The computing times are compared in [V] and it is shown that at each iteration, the IPP algorithm is able to match on the average 5.2 times as much as points without precision estimation in the same time when compared to a non-accelerated search for the closest data point in a $21 \times 21$ window around the location given by the IPP algorithm. If $\mathrm{Cov}(\mathbf{a})$ is also estimated, one iteration of the IPP algorithm with Levenberg-Marquardt is 1.7 times faster than one iteration of the ICP one for point sets with unit quaternions. The methods that could be used to accelerate the search for the closest point include k-D trees as applied in [33] or closest point caching and fast surface point computation as discussed in [28].

**Self-calibration**

The IPP algorithm for the self-calibration of the light striping system is tested in [VI]. The high accuracy and precision of the method are proved using synthetic profile maps. If the registration parameters and the intrinsic parameters of the system are assumed known, then a calibration accuracy of $0.003 \ldots 0.00003\%$ relative to the scene dimension can be achieved as the average noise level in the maps used for the calibration decreases from 0.3 down to zero pixels. For the same noise levels, the precision of the calibration parameters $(\mathrm{Tr}(\mathrm{Cov}(\mathbf{c})))^{1/2}$ ranges between $0.03 \ldots 0.0008$ and it is essentially higher than the precision of the initial calibration obtained by measuring the image coordinates of four known points. The systematic errors in $\mathbf{r}_k = [x_k \; y_k \; z_k]^T$ resulting from the errors in the calibration are of the same order of magnitude as the noise propagated from the stripe measurements on the average.

In case the registration and calibration parameters are refined simultaneously, an accuracy of $0.03 \ldots 0.09\%$ relative to the scene dimension is obtained for the noise level of 0.03 pixels in three to six maps, when the object consists of 72 planar patches in different orientations generated by triangulating a set of randomly perturbed 3-D points. The sequential iteration of the registration and calibration algorithms proves to be unsuitable since even small registration errors affect the calibration result considerably and vice versa. The accuracy figures given above and in [VI] denote the root mean squared error in $\mathbf{r}_k$ or $\mathbf{r}_{k1}$ relative to the scene dimension due to errors in the parameters estimated. In other words, it measures the distance (relative to the scene dimension in the root mean square sense without any weights) between the true $\mathbf{r}_k$ or $\mathbf{r}_{k1}$ points and the points obtained using the parameters estimated.

The test results in [VI] further show that a rather close initial calibration is needed in all cases for a successful convergence. It is also possible to refine several calibrations simultaneously if the registration is known exactly. Furthermore, the number of maps does not have to be large if the geometry of the object is appropriate within the overlapping areas. In a test case with noise-free data, the bicubic interpolation gives an order of magnitude higher accuracy than the bilinear one. A comparison to the ICP algorithm indicates the superiority of our IPP algorithm for the calibration task. An example with real data shows the qualitative improvement of the matching due to refining the calibration of the system. Further tests have indicated that a parabola fits slightly better than a straight line to the measured image coordinates of a single profile over a planar object if no camera calibration is performed. Consequently, it is likely that errors in the camera calibration have somewhat affected the matching results in the real data case in [VI, Figs. 6-7].

### Modeling

The techniques for fitting cones and cylinders are tested for different noise levels in the data using synthetic maps in [I]. Some biases can be seen in the initial estimates of the parameters of a right circular cone and cylinder if the data points cover only a part of the circumference of the quadric, but the refinement step reduces the biases considerably. The fitting works rather well even if only a small patch over the quadric is available. The real data examples illustrate the modeling of a disparity map when the scene contains planes, cones, or cylinders in [I, Figs. 2-7].

The modeling of multiple registered profile maps is shown to work qualitatively using real data in [IV, V]. This includes segmentation into planar patches, merging overlapping patches, and computing the modeled maps. An example with planar patches illustrates that the border tracing techniques can be used to view the contours of the model [IV, Fig. 6]. Further tests have indicated, however, that the switching between the domains performs poorly in this real data example.

### Combined registration and modeling

Strategies A and B for the combined registration and modeling are compared using two to six profile maps of real data in [V]. It is shown that $\mathrm{Tr}(\mathrm{Cov}(\mathbf{a}))$ and the mean of the values $\mathrm{Std}(\hat{j}_k(\mathbf{s}))$, $\mathbf{s} \in S_k$, $k = 1, \ldots, L$, remain essentially the same or get slightly better as the number of maps increases, if strategy A is used. The precision of the modeled values is also much higher than the precision of the data in areas measured with high precision.

In strategy B, the precision of the registration and the model decreases rapidly as more maps are added to the model. This results from the accumulation of errors in this strategy. When two planar patches of different maps are merged, the precision of the parameters of the resulting patch is, unfortunately, lower than the precision of the parameters of the original patches. Registration and calibration errors affect more than what is achieved by increasing the number of measurements within the patch. When the model contains merged patches of low precision more than previously, the precision of the registration of a new map against the model will usually be lower than the precision of the previous registrations. This leads to decreased precision again in the subsequent updating of the model. A solution could be to alter the criterion for merging so that two patches are merged only if the precision of the plane parameters increases after merging (see Section 7.2).

# 7 CONCLUSIONS

In this thesis, several new methods have been introduced related to the matching and modeling of multiple 3-D data sets that can be represented as real valued images on parametric domains. The major contribution deals with the iterative parametric point algorithm and its use in the registration and calibration tasks. Especially, the proposed self-calibration method is a novel contribution. The modeling techniques answer some technical details in segmentation, cylinder fitting, and border tracing. The modeling part is, however, important for the statistical analysis, which is again a substantial contribution. In the following, we itemize our contribution to knowledge and present ideas for the future research.

## 7.1 Contribution to knowledge

- We have introduced a new way of handling the data acquired by light striping in the form of a profile map, which allows us to process disparity maps and laser scanned data consistently as real valued images $\mathbf{u}_k$ on the parametric domains $S_k$, $k = 1, \ldots, L$.

- We have proposed a new and fast method to update the corresponding points on $S_k$ for the simultaneous matching of multiple maps $\mathbf{u}_k$, $k = 1, \ldots, L$, acquired from different viewpoints without exactly known corresponding points.
- We have improved the performance of the registration algorithms as regards the hierarchical processing, the interpolation errors in smooth areas, and the weighting images for the direction of the surface normal, for the edge areas, and for the precision of the distance between the corresponding points.
- We have experimentally shown that for the registration of profile maps, the IPP algorithms with bicubic and bilinear interpolation give more accurate results and have larger pull-in ranges than the ICP ones for triangle sets, for bilinearly interpolated quadrilaterals, and a variation of the ICP one minimizing the distance from the points of a data set to tangent planes at the points of intersection between the surface normals of the data set and a bilinearly interpolated surface constructed from the points of the other data set.
- We have proposed a new and accurate method for the self-calibration of the light striping system based on matching multiple profile maps and using the IPP algorithm.
- We have developed a new technique for the initial estimation of the parameters of a cylinder from $\mathbf{u}_k$.
- We have presented techniques for the modeling of multiple maps $\mathbf{u}_k$ including the splitting and merging of compatible overlapping planar patches of different maps and the tracing of the border of a merged patch on multiple domains $S_k$, $k = 1, \ldots, L$.
- We have carefully analyzed the propagation of measurement and calibration errors to the registration parameters and to the modeled maps and thereby derived the precision estimates for two combined registration and modeling strategies.
- We have shown in a test case that our statistical analysis yields higher precision for the strategy where the maps are first registered and the model is built afterwards than for the strategy where the maps are registered against the model updated whenever a new map has been registered.
- We have demonstrated how most of the developed algorithms can be formulated using image algebra and implemented efficiently using MATLAB software.
- We have proved the performance of the algorithms developed in various experiments with synthetic and real data and developed the techniques needed for generating synthetic profile maps.

## 7.2 Future research

During this work, we have identified the following unsolved questions that are beyond the scope of the thesis.

The future development of the matching algorithm could be focused on edge areas. All the locations near edges are currently disregarded or weighted according to a decay function in order to avoid large interpolation errors that may cause systematic errors to the parameters estimated in the matching. Since edge areas may contain useful data for the matching, the magnitude of the interpolation errors could be estimated using higher order derivatives and the corresponding points weighted accordingly. The interpolation errors could also be reduced increasing the density of profiles and image observations in edge areas.

There exist closed form solutions to estimating the parameters of a rigid body transformation between two 3-D data sets when the corresponding points between the sets are known, but the case of estimating simultaneously multiple transformations between multiple sets in a closed form is unsolved.

The self-calibration method could be extended to the stereo vision system. In this thesis, we have preferred profile maps to disparity maps in the real data tests since light striping provides 3-D data directly after calibration while the additional step of correspondence matching is required in stereo. Parameters for correcting the lens distortion of the camera could also be included in the self-calibration method.

The IPP algorithm has been developed for data sets that can be represented as single valued images where only one measured coordinate contains noise. The method could be generalized to multi valued images. The corresponding points would be determined on the parametric domains as before and the distance would be defined using the metric of the value set of the images. Especially, this would extend the method to measuring systems where each of the $x, y, z$ coordinates is corrupted by noise independent of the other coordinates.

A connected surface model of the object could be generated from the modeled maps. This would require finding out the adjacency between the patches of different maps and dealing with occluding edges. Related works on the integration problem are referred to in [IV].

The accuracy of the modeled maps could be estimated using synthetic data. The border tracing techniques including the switching between the maps could be tested using synthetic data and also real data after refining the calibration of the system. It could be investigated how much the switching depends on the accuracy of the registration.

The tracing techniques provide the outlines of the surface model. These could be utilized in interactive wire frame modeling for virtual reality [15]. The related research on reconstructing a wire frame model from multiple views is discussed in [IV]. We consider also CAD interfacing in [1, 20].

The different registration and modeling strategies could be compared to each other regarding the accuracy obtained. In the precision estimation, we currently assume that the calibration has been solved in advance. The analysis could be extended to the self-calibration method, where the registration and calibration parameters depend on each other. This would mainly mean that $\mathbf{a}$ should be replaced by $[\mathbf{a}^T\ \mathbf{c}^T]^T$ in the derivation for the case of a simultaneous registration and calibration. It could also be investigated how accurately the maps should be aligned in order that the precision of the parameters of a merged planar patch would be higher than the precision of the parameters of the original patches of different maps.

The techniques for generating synthetic profile maps could be utilized in other applications such as planning convenient scanning paths for object digitization if, e.g., a CAD model of the object is available. New measurements could be acquired from areas measured with low precision. The precision of the reconstructed model could be analyzed to solve the next best view problem as in [32]. The difference images between the measurements and the model could verify whether the type of the model primitive was correctly selected.

Although we are using sparse matrices, our MATLAB programs need quite a lot main memory of the computer. The algorithms could be converted into c or c++ using the built-in compiler of MATLAB. The memory savings would make the investigation of the registration and modeling strategies more attractive as more maps could be used.

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# CORRECTIONS TO THE PUBLICATIONS

The following errors have been found in the publications. There are also some grammatical and typographical mistakes as there has been no proof reading of the language in [I, II, IV, VI].

### Paper [I]

- p. 594, 4th paragraph, line 5: "[16]" should read "[15]"
- p. 596, 3rd paragraph, line 8: "[2] and [12]" should read "[12]"
- p. 597, 2nd paragraph, lines 6-7: "first making rotations of $\theta$ and $\varphi$ about the $X$ and $Y$ axes, respectively, and then a translation of the origin to $(X_0, Y_0, Z_0)$." should read "first making a translation of the origin to $(X_0, Y_0, Z_0)$ and then rotations of $\theta$ and $\varphi$ about the $X'$ and $Y''$ axes, respectively."
- p. 597, Eq. (2.1): "$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$", "$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$", and "$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix}$" should read "$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$", "$\begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix}$", and "$\begin{pmatrix} X''' \\ Y''' \\ Z''' \end{pmatrix}$", respectively
- p. 597, 3rd paragraph, lines 5-6: "first performing rotations according to (2.1) and then a translation of the rotation centre, i.e., the origin, to $(X_0, Y_0, Z_0)$." should read "first making a translation of the origin to $(X_0, Y_0, Z_0)$ and then performing rotations of $\theta$ and $\varphi$ according to (2.1)."

### Paper [II]

- p. 158, 2nd paragraph, line 3: "coordinate transformation" should read "rigid body coordinate transformation"
- p. 159, 5th paragraph, line 5: "smaller" should read "not larger"
- p. 159, 5th paragraph, line 5: "larger" should read "not smaller"

### Paper [III]

- p. 436, 5th paragraph, line 2: "$\mathbf{s}_{kl} \in O_{kl}^{(l)}$" should read "$\mathbf{s}_k \in O_{kl}^{(l)}$"

### Papers [III−V]

Two errors in the implementation of the algorithms have been found. The first one deals with computing the derivatives in the Levenberg-Marquardt algorithm. They have been evaluated partly in a false point and this has affected the registration results in [III, IV], but it has been corrected in [V]. The second one deals with the direction of the surface normal when its $z$ component is near zero in the $x, y, z$ coordinate system of the light striping system. In some locations, the normal has been an inner normal although we have intended to use the outer one. This has made the registration more difficult in [III—V] mainly since some of the corresponding points have been rejected as incompatible.

**Paper [IV]**

- p. 245, 5th paragraph, line 3: “overlapping area” should read “overlapping area transformed onto $S_l$”
- p. 245, 7th paragraph, lines 1-2: “$\mathbf{s}_l$ on $S_l$ satisfying $c_l(\mathbf{s}_l) = c_l(\langle \mathbf{s}_{kl}^{\gamma} \rangle)$” should read “$\mathbf{s}_l = \langle \mathbf{s}_{kl}(\mathbf{s}_k) \rangle$ on $S_l$ satisfying $c_l(\langle \mathbf{s}_{kl}(\mathbf{s}_k) \rangle) = c_l(\langle \mathbf{s}_{kl}^{\gamma} \rangle)$ and $c_k(\mathbf{s}_k) = \gamma_k$ inside the overlapping area on $S_k$, and $\mathbf{s}_l$ on $S_l$ satisfying $c_l(\mathbf{s}_l) = c_l(\langle \mathbf{s}_{kl}^{\gamma} \rangle)$ outside the overlapping area transformed onto $S_l$”

**Paper [V]**

- p. 321, 5th paragraph, line 3: “corresponding” should read “intersected”
- p. 323, 3rd paragraph, line 2: “$\mathrm{Var}(\mathbf{y}) = 0$” should read “$\mathrm{Var}(\mathbf{y}) = \mathbf{0}$”
- p. 323, 3rd paragraph, line 4: “$\mathrm{Var}(j(s))$” should read “$\mathrm{Var}(j(\mathbf{s}))$”
- p. 324, 4th paragraph, line 19: “ICP algorithm” should read “ICP algorithm for point sets”
- p. 324, 4th paragraph, line 19: “$(\mathbf{s}_k, \mathbf{r}_k, (\mathbf{s}_k))$” should read “$(\mathbf{s}_k, \mathbf{r}_k(\mathbf{s}_k))$”
- p. 326, 3rd paragraph, line 4: “block diagonal” should be removed
- p. 327, 2nd paragraph, line 18: “$\mathbf{Cov}(\mathbf{a}) = \mathbf{Cov}(\mathbf{\Delta a})$” should read “$\mathrm{Cov}(\mathbf{a}) = \mathrm{Cov}(\mathbf{\Delta a})$”
- p. 327, 3rd paragraph, line 4: “$\hat{j}_l$” should read “$\tilde{j}_l$”
- p. 333, 2nd paragraph, line 3: “ICP algorithm” should read “ICP algorithm for point sets”

**Paper [VI]**

- p. 180, 2nd paragraph, lines 15-16: “closest points” should read “closest data points”
- p. 187, 3rd paragraph, line 6: “closest points” should read “closest data points”
- p. 187, 3rd paragraph, lines 11-12: “since the results were better without it” should be removed
- p. 188, 1st paragraph, line 13: “works best and” should be removed
- p. 189, 3rd paragraph, line 7: “standard ICP algorithm” should read “ICP algorithm for point sets”

# Paper [I]

Jokinen, O., Reconstruction of quadric surfaces from disparity measurements.

# Paper [II]

Jokinen, O. and Haggrén, H., Relative orientation of two disparity maps in stereo vision.

## Paper [III]

Jokinen, O., Area-based matching for simultaneous registration of multiple 3-D profile maps.

# Paper [IV]

Jokinen, O., Building 3-D city models from multiple unregistered profile maps.

## Paper [V]

Jokinen, O. and Haggrén, H., Statistical analysis of two 3-D registration and modeling strategies.

# Paper [VI]

Jokinen, O., Self-calibration of a light striping system by matching multiple 3-D profile maps.

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